

Math 45 SSM 2/e 3.4 Slope-Intercept Form of a Line

Objectives:

- 1) Discover slope and y-intercept locations in the slope-intercept form of the equation of a line
- 2) Use algebra to find slope and y-intercept
- 3) Graph lines in $y=mx+b$ form and $Ax+By=C$ form using slope and y-intercept.
- 4) Write the equation of a line given the slope and y-intercept.
- 5) Linear models in slope-intercept form.

yes ① Sketch graph of $4x+2y=10$.

a) Find y-intercept

b) Find slope

c) Solve $4x+2y=10$ for y. to write in slope-intercept form.

a) y-intercept: set $x=0$ and solve for y

$$4(0) + 2y = 10$$

$$0 + 2y = 10$$

$$\frac{2y}{2} = \frac{10}{2}$$

$$y = 5$$

$$\boxed{y_{int} (0, 5)}$$

b) Find another point — choose a value for x.

$$x = 1$$

$$4(1) + 2y = 10$$

$$4 + 2y = 10$$

$$\begin{array}{r} -4 \\ 4 + 2y = 10 \\ \hline \end{array}$$

$$\frac{2y}{2} = \frac{6}{2}$$

$$y = 3$$

$$(1, 3)$$

slope formula on $(1, 3)$ and $(0, 5)$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{5 - 3}{0 - 1}$$

$$= \frac{2}{-1}$$

$$\boxed{m = -2}$$

← Note:
This is the long way.
Soon we'll know a shorter way

c) $4x + 2y = 10$

$$\begin{array}{r} -4x \\ 4x + 2y = 10 \\ \hline \end{array}$$

$$\frac{2y}{2} = \frac{-4x + 10}{2}$$

$$\boxed{y = -2x + 5}$$

slope is here
(coefficient of x in $y=mx+b$)

y-coordinate of y-intercept
is here (constant in $y=mx+b$)

no ② Sketch $3x - 2y = 6$

- Find y-int
- Find slope
- Solve to write in slope-int form

a) y-int: set $x=0$ and solve for y

$$3(0) - 2y = 6$$

$$0 - 2y = 6$$

$$\frac{-2y}{-2} = \frac{6}{-2}$$

$$y = -3$$

$(0, -3)$ y-int

b) Another point: set $y=0$.

$$3x - 2(0) = 6$$

$$3x - 0 = 6$$

$$\frac{3x}{3} = \frac{6}{3}$$

$$x = 2$$

$(2, 0)$ another point

slope formula on $(0, -3)$ and $(2, 0)$
 (x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{0 - (-3)}{2 - 0}$$

$$m = \frac{3}{2}$$

Note:
Still the long way!

c) Isolate y:

$$\begin{array}{rcl} 3x - 2y & = & 6 \\ -3x & & -3x \end{array}$$

$$\frac{-2y}{-2} = \frac{-3x + 6}{-2}$$

$$y = \frac{3}{2}x - 3$$

Slope is coefficient of x

y-int is constant term

CAUTION:

only if equation is written in slope-intercept form!

Slope-Intercept Form of equation of a line

$$y = mx + b$$

where m is slope and b is y-intercept

Find the slope and y-intercept.

← Now, we're doing it the short way.

NO (3) $y = 4x - 3$

step 1: Equation is already in slope intercept form.

step 2:	$\text{slope} = 4$ $y\text{-int} = (0, -3)$	$m = 4$ $b = -3$
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NO (4) $3x + y = 7$

step 1: Solve for y, write in $y = mx + b$ form.

$$\begin{array}{r} 3x + y = 7 \\ -3x \quad -3x \end{array}$$

$$y = -3x + 7$$

step 2: constant is y-int $(0, 7)$
coefficient of x is slope $m = -3$

CAUTION: Slope is a number, not a variable.
x is not part of slope

YES (5) $2x + 5y = 15$

step 1: solve for y

$$\begin{array}{r} 2x + 5y = 15 \\ -2x \quad -2x \end{array}$$

$$\frac{5y}{5} = -\frac{2x}{5} + \frac{15}{5}$$

$$y = -\frac{2}{5}x + 3$$

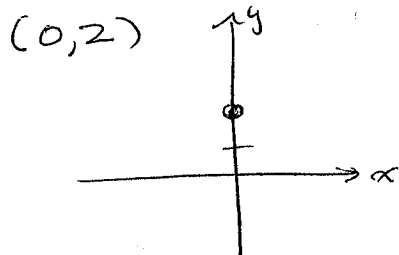
step 2:

$y\text{-int } (0, 3)$ $\text{slope } m = -\frac{2}{5}$
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Graph using the slope and y-intercept.

NO ⑥ $y = \frac{4}{3}x + 2$

Step 1: Identify and plot the y-intercept

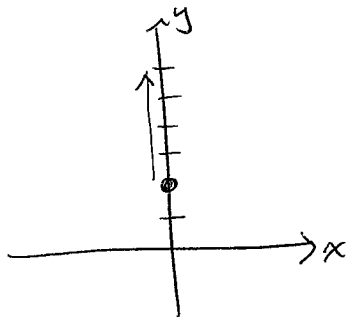


Step 2: Identify the slope and write as $\frac{\text{rise}}{\text{run}}$.

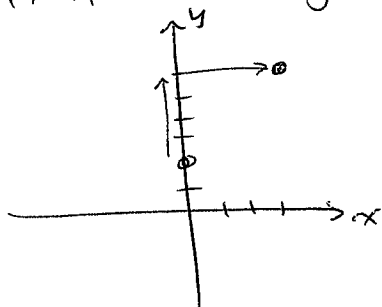
$$m = \frac{4}{3} = \frac{\text{rise}}{\text{run}}$$

rise 4 units up
run 3 units right

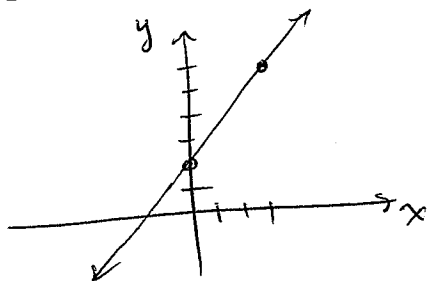
Step 3: Start from the y-intercept and go "rise" units up
(or down if rise is negative)



Step 4: From end of arrow in step 3 go "run" units right
(or left if run is negative) and plot another point on line



Step 5: Connect the dots, extend graph to edge of grid.



Repeat steps 3 & 4
from any point
on the line if needed

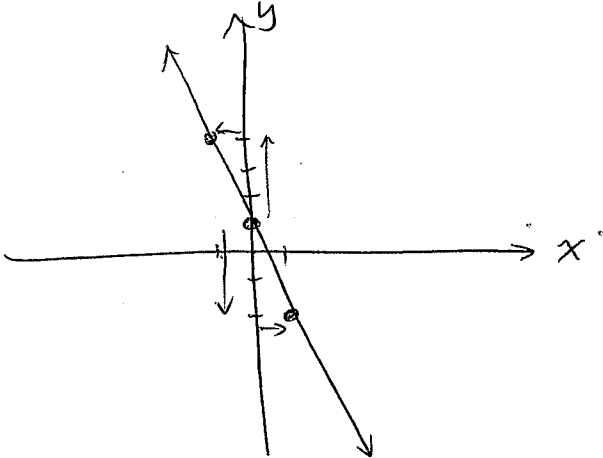
M45 SSM 2/e 3.4 p.5

Graph using slope and y-intercept.

NO ⑦ $y = -3x + 1$

y-int $(0, 1)$

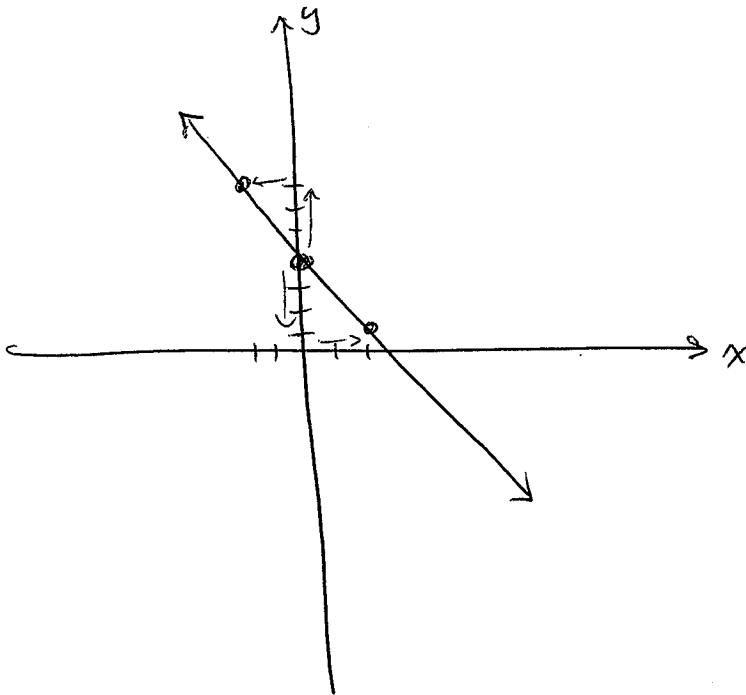
slope $m = -3 = \frac{-3}{1} = \frac{\text{rise}}{\text{run}}$ OR $\frac{3}{-1} = \frac{\text{rise}}{\text{run}}$



YES ⑧ $y = -\frac{3}{2}x + 4$

y-int $(0, 4)$

slope $m = -\frac{3}{2} = \frac{\text{rise}}{\text{run}}$ OR $\frac{3}{-2} = \frac{\text{rise}}{\text{run}}$



M45 SSM 2/e 3.4 p.6

Graph using slope and y-intercept.

yes ⑨ $-2x + y = -3$

step 1: write in $y = mx + b$ form

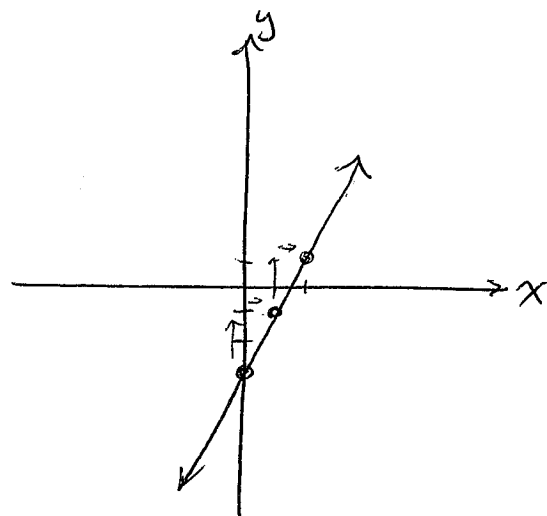
$$\begin{array}{r} -2x + y = -3 \\ +2x \quad \quad +2x \end{array}$$

$$y = 2x - 3$$

step 2 continue as before

y-int $(0, -3)$

slope $m = 2 = \frac{2}{1} = \frac{\text{rise}}{\text{run}}$



no ⑩ $6x - 2y = 2$

step 1: write in $y = mx + b$ form

$$\begin{array}{r} 6x - 2y = 2 \\ -6x \quad \quad -6x \end{array}$$

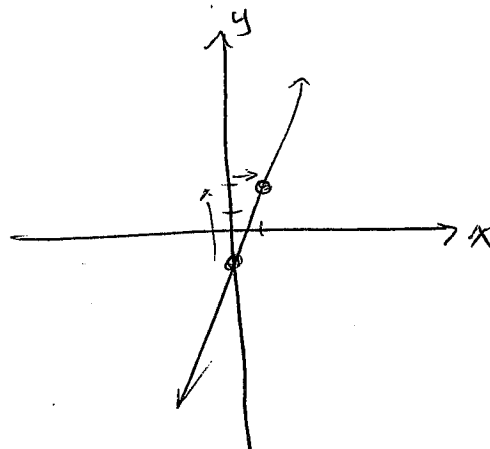
$$\begin{array}{r} -2y = -6x + 2 \\ -2 \quad \quad -2 \quad \quad -2 \end{array}$$

$$y = 3x - 1$$

step 2: continue as before

y-int $(0, -1)$

slope $m = 3 = \frac{3}{1} = \frac{\text{rise}}{\text{run}}$



yes ⑪ $3x + 5y = 0$

$$\begin{array}{r} 3x + 5y = 0 \\ -3x \quad \quad -3x \end{array}$$

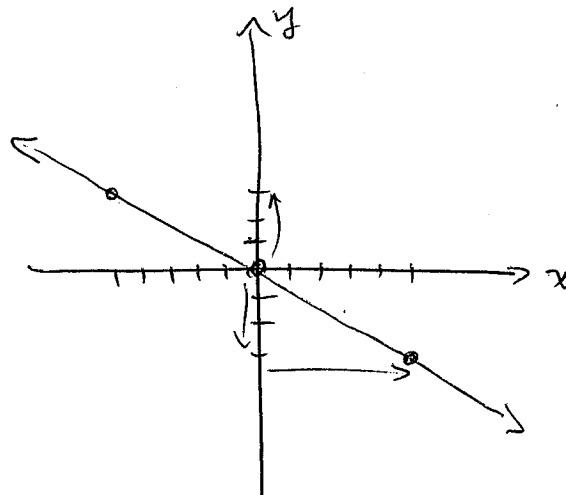
$$\frac{5y}{5} = \frac{-3x}{5}$$

$$y = -\frac{3}{5}x$$

y-intercept is 0! $(0, 0)$

slope $m = -\frac{3}{5} = \frac{\text{rise}}{\text{run}}$

or $m = \frac{+3}{-5} = \frac{\text{rise}}{\text{run}}$



Write the equation of the line with given characteristics.

YES (12) slope $\frac{3}{8}$, y-int -4

$$y = mx + b$$

$$y = \frac{3}{8}x + (-4)$$

$$\boxed{y = \frac{3}{8}x - 4}$$

substitute $m = \frac{3}{8}$

and $b = -4$

into $y = mx + b$
and simplify.

NO (13) slope 3, y-int -2

$$y = mx + b$$

$$\boxed{y = 3x - 2}$$

NO (14) slope $-\frac{1}{4}$, y-int 3

$$y = mx + b$$

$$\boxed{y = -\frac{1}{4}x + 3}$$

YES (15) slope 0, y-int -1

$$y = mx + b$$

$$y = 0x - 1$$

$$\boxed{y = -1}$$

(16) A woman's total cholesterol y is related to her age x by the equation $y = 1.1x + 157$

a) Predict cholesterol for a woman age 40.

$$\text{age} = x \Rightarrow \text{subst } x = 40$$

$$y = 1.1(40) + 157$$

$$\boxed{y = 201}$$

b) Determine and interpret slope

$$\boxed{m = 1.1}$$

cholesterol rises 1.1 units each year

c) Determine and interpret y-intercept.

at $x = 0$, $y = 157$ means an infant age 0 has cholesterol 157 units

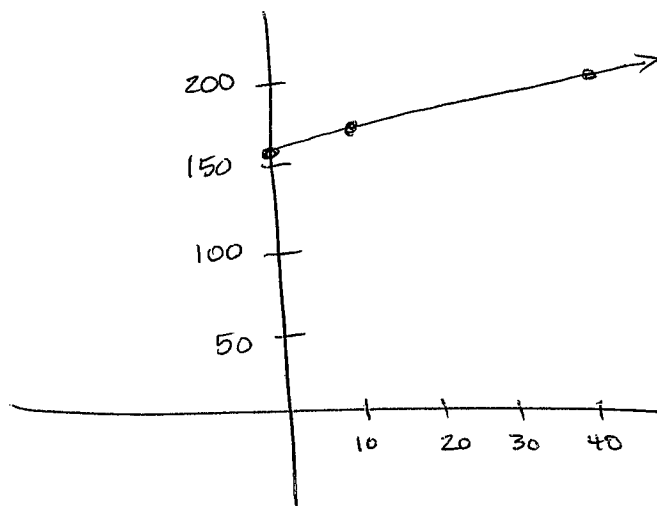
d) Graph the equation

cont $\rightarrow$

(16) cont

d) $y = 1.1x + 157$

Table	x	y
	0	157
	10	168
	40	201



(17) The annual cost of operating a Chevy Cobalt is \$0.25 per mile plus \$3000.

a) Write a linear equation that relates cost y to miles driven x .

$$\left\{ \begin{array}{l} \text{drive no miles } x=0, \text{ cost } \$3000 = y \\ \text{y-int } (0, 3000) \\ b=3000. \end{array} \right.$$

$$\left\{ \begin{array}{l} \$0.25 \text{ per mile is a rate} = \text{slope is rate} \\ m = 0.25 \end{array} \right.$$

$$\boxed{y = 0.25x + 3000}$$

b) What is cost of driving 11,000 miles?

$$\text{miles} = x = 11,000$$

$$\text{subst } x = 11,000$$

$$y = 0.25(11,000) + 3000$$

$$\boxed{y = \$5750}$$

c) Graph the equation

Table	x	y
	0	3000
	11,000	5750

